Prediction of Heat Transfer in Turbulent Falling Liquid Films with or without Interfacial Shear

A unified approach is proposed for the prediction of heat transfer coefficients in turbulent falling films undergoing heating, evaporation or condensation for both of the cases with or without interfacial shear. A modified van Driest eddy viscosity model, which incorporated a damping factor f and takes into account the effect of variable shear stress, is used to predict the hydrodynamics of turbulent falling films. The calculated film thicknesses are in good agreement with the Nusselt-Brauer correlations for the non-sheared film and the Dukler prediction for highly sheared film. Also, by including a van Driest type turbulent Prandtl number model, the asymptotic heat transfer coefficients are accurately predicted and show better agreement with the extensive literature data and correlations than do most of the existing turbulence models proposed to date.

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SCOPE

A number of experiments have been performed on turbulent film heating, evaporation and condensation for both of the cases with or without interfacial shear. The prediction of average film thickness and heat transfer coefficients for these processes usually requires a specification of a turbulence model for the eddy diffusivity of momentum and heat as derived from single-phase flow. Although there are several turbulence models proposed in the literature, they are often limited to describing just a certain heat transfer process. Therefore, the applicability of these models to all of the above-mentioned processes is questionable.

The present research proposes a simple, unified procedure for the prediction of average film thicknesses and asymptotic heat transfer coefficients for these processes based on a single turbulence model. The model used is a modified van Driest eddy viscosity model which incorporates a damping factor f and takes into account the effect of interfacial shear through the variable shear stress term. A van Driest type turbulent Prandtl number model with variable shear stress is also included to account for the difference in the eddy diffusivities of momentum and heat. Verification of the model is established by comparing the predictions with extensive literature data from various film heat transfer processes.

CONCLUSIONS AND SIGNIFICANCE

A modified van Driest eddy viscosity model, which incorporates a damping factor f and takes into account the effect of variable shear stress, is developed. The model can, on the average, predict satisfactorily the average film thicknesses and asymptotic heat transfer coefficients for film heating, evaporation and condensation in the turbulent flow region of $Re >$ 2,400 for both with or without interfacial shear. The computations from the model show better agreement with the extensive literature data and correlations than most of the existing turbulence models proposed to date. Thus, interpretation of the data obtained from the above industrially important film heat transfer processes can be made more simply on a unified basis.

The falling film shell and tube heat exchanger is widely used in the process heat transfer industry in heating, cooling, evaporation and condensation. These processes are usually operated in the turbulent flow regime to increase the capacity and rate of heat transfer. Data for film heating were obtained by McAdams et al. (1940), Wilke (1962), Gimbutis (1974), Ishigai et al. (1974), Ueda and Tanaka (1974), and Fujita and Ueda (1978a). Film evaporation data were presented by Chun (1969), Chun and Seban (1971), Taubman and Kalishevich (1976), and Fujita and Ueda (1978b). Film condensation data were reported by Carpenter (1948), Kutateladze (1963), Ueda et al. (1972), Ueda et al. (1974), Blangetti and Schlunder (1978), and Kutateladze and Gogonin (1979).

The prediction of heat transfer coefficients for these processes is not an easy task, especially in the presence of significant interfacial shear exerted by the vapor. Various turbulent models have been proposed, Table 1. Some models have shown partial success with regard to the particular heat transfer process investigated, but it is questionable whether they can be applied equally well to other processes of a similar nature. The main difficulty seems to lie in the proper specification of an unique turbulence model that can be generally used for all these processes. A desirable turbulence model should give good prediction of both the average film thickness and the heat transfer coefficient with or without the presence of interfacial shear.

In this work, we propose a unified procedure for the prediction of average film thicknesses and asymptotic heat transfer coeffi-

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TABLE 1. IMPORTANT TURBULENCE MODELS FOR PREDICTION OF HEAT TRANSFER IN FALLING LIQUID FILMS

Author	Process	Turbulence Model Used	Comparison with Data
Seban 1954	Condensation	Prandtl-Nikuradse Model; $\epsilon_M/\nu = 0 \quad 0 \leqslant y^+ \leqslant 5$ $\epsilon_M/\nu = y^+/5 - 1 \quad 5 \leqslant y^+ \leqslant 30$ $\epsilon_M/\nu = y^+/2.5 \quad 30 \leqslant y^+$ $Pr_t = 1$	h_m^* agrees with Colburn's analogy results for $Pr = 5$; h_m^* lower than data by 10 to 15% for $2 < Pr < 5$, $Re < 40{,}000$.
Rohsenow, Webber and Ling (1956)	Condensation with Interfacial Shear		h_m^* agrees fairly with the data of Carpenter and Colburn for $2 < Pr < 3$, $5 < \tau_i^* < 50$.
Dukler (1960, 1961 a,b)	Condensation with Interfacial Shear	Deissler model for $y^+ \leqslant 20$; $\epsilon_M/\nu = (0.125)^2 u^+ y^+ (1 - \exp(-(0.125)^2 u^+ y^+))$; von Karman model for $y^+ > 20$; $\epsilon_M/\nu = (0.4)^2 (du^+/dy^+)^3/(d^2u^+/dy^{+2})^2$; $Pr_t = 1$	δ^* vs. Re slightly higher than data for $0 \leqslant \tau_i^* \leqslant 200$; h_m^* agrees fairly well with Carpenter's data (1948).
Mills and Chung (1973)	Evaporation	Original van Driest model for inner region; $\epsilon_M/\nu = -0.5 + 0.5\{1 + 0.64 y^{+2} (1 - \exp(-y^+/26))^2\}^{1/2}$; Modified Lamourelle and Sandall (1972) gas absorption eddy diffusivity for outer region; $\epsilon_M/\nu = 6.47 \times 10^{-4} (\rho g^{1/3} \nu^{4/3}/\sigma \delta^{+2/3}) (\sigma^+ - y^+)^2 \cdot Re^{1.678}$; $Pr_t = 0.9$	δ^+ vs. Re higher than Brauer's correlation by 15% as shown by Seban and Faghri (1976); h_x^* agrees well with data of Chun and Seban (1971).
Limberg (1973)	Heating	Modified van Driest model for $y/\delta \leqslant 0.6$, $\epsilon_M/\nu = -0.5 + 0.5\{1 + 0.64 y^{+2} (\tau/\tau_w)\{1 - \exp(-y^+, (\tau/\tau_w)^{1/2}/25.1)\}^2 f^2\}^{1/2}$ $\tau/\tau_w = 1 - y^+/\delta^+$; $f = \exp(-1.66 y^+/\delta^+)$; $\epsilon_M/\nu = \epsilon_M/\nu\|_{y/\delta=0.6}$ = constant for $0.6 < y/\delta \leqslant 1$; $Pr_t = 0.89$	δ^+ vs. Re conform to Brauer's correlation as shown by Seban and Faghri (1976); Nu_x versus Re slightly higher than Wilke's data (1962) at high Re but much higher than Wilke's data at low Re.
Ishigai, Nakanisi, Takehara, and Oyabu (1974)	Heating	Spalding model for all y^+; $\epsilon_M/\nu = 0.1108 B\{\exp(Bu^+) - 1 - Bu^+ - (Bu^+)^2/2 - (Bu^+)^3/6\}$ $Pr_t = 1$, $B = 0.4$	δ^* vs. Re 10 to 20% higher than Brauer's correlation; Nu_x vs. Re slightly higher than Wilke's data (1962) at high Re but much higher than Wilke's data at low Re
Ueda, Kubo and Inoue (1974)	Condensation with Interfacial Shear	Rohsenow, Webber and Ling's model $Pr_t = 1$	δ^* vs. Re slightly higher than data for $0 \leqslant \tau_i^* \leqslant 200$; h_x^* vs. Re higher than data.
Hubbard, Mills and Chung (1976)	Condensation, Evaporation with Interfacial Shear	Modified van Driest model for inner region, $\epsilon_M/\nu = -0.5 + 0.5\{1 + 0.64 y^{+2} (\tau/\tau_w)\{1 - \exp(-y^+/26)\}^2\}^{1/2}$ $\tau = \rho g(\delta - y) + \tau_i$ Empirical Chung's (1974) gas absorption eddy diffusivity for outer region, $\epsilon_M/\nu = 8.13 \times 10^{-17} \{(\nu g)^{2/3}/Cbv^{*2}\} Re^{2n} \{1 + b(\tau_i/\tau_w)\}^2 (\delta^+ - y^+)^2$ $Pr_t = 0.9$	slope of h_x^* vs. Re curve lower than slope of data curve of Chun and Seban (1971) for evaporation, h_x^* vs. Re for condensation higher than data of Ueda et al. (1974) for $Pr = 2$ and $10 \leqslant \tau_i^* \leqslant 200$.
Seban and Faghri (1976), Faghri (1976)	Evaporation and Heating	Modified van Driest model proposed by Limberg (1973) for $y/\delta \leqslant 0.6$ ϵ_M/ν = same as Limberg for $y/\delta \leqslant 0.6$; $\epsilon_M/\nu = \epsilon_M/\nu\|_{y/\delta=0.6}$ = constant for $0.6 < y/\delta \leqslant y_e/\delta$; Modified Lamourelle and Sandall (1972) gas absorption eddy diffusivity as proposed by Mills and Chung (1973) for $y_e/\delta < y/\delta \leqslant 1$; ϵ_M/ν = same as Mills and Chung for outer region; $Pr_t = 0.9$ (Model 3A); $Pr_t = 0.9\{1 - \exp(-y^+/A^+)\}/\{1 - \exp(-y^+/B^+)\}$ (Model 3)	δ^+ vs. Re conform to Brauer's correlation; h_x^* vs. Re agree with evaporation data of Chun and Seban (1971) for $Pr = 1.77$ but lower than the data for $Pr = 5.7$; h_x^* vs. Re slightly higher than Wilke's correlation for heating (1962) at high Re but much higher than the correlation at low Re for $Pr = 5.4$.
Razavi and Damle (1978)	Condensation with Interfacial Shear	Deissler model for $y^+ \leqslant 26$ as proposed by Dukler (1960); ϵ_M/ν = same as Duckler for $y^+ \leqslant 26$; von Karman model for $y^+ > 26$; ϵ_M/ν = same as Dukler for $y^+ > 26$; $Pr_t = 1$, momentum change of condensing vapor included in evaluating interfacial shear	h_x^* vs. Re agrees better with data of Carpenter (1948) and Goodykoontz and Dorsch (1966, 1967) than the predictions of Nusselt, Dukler (1960), Kunz and Yerazunis (1969), Shekriladze and Mestvirishvili(1973) as cited by Razavi and Damle (1978)

cients in turbulent film heating, evaporation or condensation for both with or without interfacial shear. The turbulence model used in the present work incorporates the main features of the van Driest model used by Limberg (1973), Seban and Faghri (1976) and Hubbard et al. (1976), but with two important differences. The first difference is the inclusion of the effect of interfacial shear through the variable shear stress term in the van Driest eddy viscosity model and the turbulent Prandtl number model. The second difference is that an interface damping eddy diffusivity model as deduced from gas absorption data of Lamourelle and Sandall (1972) or Chung (1974) is not used for the gas-liquid interface region of film heating, evaporation or condensation for the following two reasons:

(1) it is not certain whether an empirical eddy diffusivity as derived from the large liquid Schmidt number gas absorption experiments can be used for the much lower liquid Prandtl number heat transfer predictions.

(2) the thickness of this damped turbulence region in which the empirical eddy diffusivity is assumed to apply is unknown because it varies with the turbulence and physical properties as discussed by Yih (1981).

Certain arbitrariness have been called upon in specifying the thickness of this region as have been done by Mills and Chung (1973), Hubbard et al. (1976), and Seban and Faghri (1976). Until more information is gathered on understanding the nature of interface damping and predicting the thickness of this region, the specification of an eddy diffusivity as deduced from gas absorption data for the interface region of film heating, evaporation or condensation seems to be quite unjustified and unnecessary.

THEORY

Hydrodynamics

The total shear stress distribution in the liquid film is given by

$$\tau = \rho(\delta - y)g/g_c + \tau_i \tag{1}$$

where τ_i is the interfacial shear. The wall shear is

$$\tau_w = \rho\delta g/g_c + \tau_i \tag{2}$$

If we define $\delta^* = \delta/(\nu^2/g)^{1/3}$, $\delta^+ = \delta u^*/\nu$, $u^* = (\tau_w g_c/\rho)^{1/2}$,

$$\tau_i^* = \tau_i g_c/\rho(\nu g)^{2/3},\ s^3 = (\rho\delta g/g_c)/(\tau_i + \rho\delta g/g_c),$$

becomes

$$s^3 + \tau_i^* s^2/\delta^{+2/3} - 1 = 0 \tag{3}$$

and $\delta^* = s\delta^{+2/3}$. The universal velocity distribution is given by

$$\frac{du^+}{dy^+} = \frac{1 - s^3 y^+/\delta^+}{1 + \epsilon_M/\nu} \tag{4}$$

where $\tau/\tau_w = 1 - s^3y^+/\delta^+$, $u^+ = u/u^*$, $y^+ = yu^*/\nu$, and ϵ_M is the eddy diffusivity for momentum. If $\tau_i = 0$, $s^3 = 1$ and the above equations reduce to the nonsheared film case. The film Reynolds number is calculated from

$$Re = 4\Gamma/\mu = 4\int_0^{\delta^+} u^+ dy^+ \tag{5}$$

where Γ is the mass flow rate of liquid per unit periphery.

Heating at Constant Wall Heat Flux

The energy equation is represented, in dimensionless form, by

$$u^+ \frac{\partial\theta}{\partial x^*} = \frac{\partial}{\partial\eta}\left(\left(1 + \frac{\epsilon_H}{\nu}Pr\right)\frac{\partial\theta}{\partial\eta}\right) \tag{6}$$

where $\theta = (T - T_{in})/(q_w\delta/k)$, $x^* = x\alpha/\delta^2 u^*$, $\eta = y^+/\delta^+$, and ϵ_H is the eddy diffusivity for heat. The boundary conditions are

$x^* = 0$ (inlet)	$\theta = 0\ (T = T_{in})$	(7)
$\eta = 0$ (wall)	$\dfrac{\partial\theta}{\partial\eta} = -1$ (constant wall heat flux)	(8)
$\eta = 1$ (gas-liquid interface)	$\dfrac{\partial\theta}{\partial\eta} = 0$ (zero interfacial heat flux)	(9)

When the velocity and temperature profiles are fully developed, $\partial\theta/\partial x^* = d\theta_m/dx^* =$ constant where θ_m is the dimensionless bulk average temperature. Equation 6 can be integrated to give

$$\theta - \theta_w = \frac{4\delta^+}{Re}\int_0^\eta \frac{\int_0^\eta u^+ d\eta - Re/4\delta^+}{(1 + \epsilon_H Pr/\nu)}d\eta \tag{10}$$

where θ_w is the dimensionless wall temperature. The heat transfer coefficient is defined as $h_x = q_w/(T_w - T_m)$ and the Nusselt number can be derived as

$$Nu_x = \frac{1}{(\theta_w - \theta_m)} = \left(\frac{16\delta^{+2}}{Re^2}\int_0^1 u^+ \times\left(\int_0^\eta \frac{Re/4\delta^+ - \int_0^\eta u^+ d\eta}{(1 + \epsilon_H Pr/\nu)}d\eta\right)d\eta\right)^{-1} \tag{11}$$

The dimensionless heat transfer coefficient h_x^* is related to Nu_x by

$$h_x^* = (h_x/k)(\nu^2/g)^{1/3} = Nu_x/s\delta^{+2/3} \tag{12}$$

Evaporation or Condensation at Constant Wall Heat Flux

The energy equation is still represented by Eq. 6 but θ is now defined as $\theta = (T - T_{sat})/(q_w\delta/k)$ with the boundary conditions:

$x^* = 0$ (inlet)	$\theta = 0\ (T = T_{sat})$	(13)
$\eta = 0$ (wall)	$\dfrac{\partial\theta}{\partial\eta} = -1$ (constant wall heat flux)	(14)
$\eta = 1$ (gas-liquid interface)	$\theta = 0\ (T = T_{sat})$	(15)

Under fully developed conditions, $\partial\theta/\partial x^* = 0$. Equation 6 can be integrated to give

$$\theta = \int_\eta^1 \frac{d\eta}{1 + \epsilon_H Pr/\nu} \tag{16}$$

The heat transfer coefficient is defined as $h_x = q_w/(T_w - T_{sat})$. The Nusselt number can be derived as

$$Nu_x = 1/\theta_w = 1\Big/\int_0^1 \frac{d\eta}{1 + \epsilon_H Pr/\nu} \tag{17}$$

Although the preceding development is for the boundary condition of constant wall heat flux, the resulting asymptotic Nusselt number should apply equally well to the boundary condition of constant wall temperature because, for turbulent flow with Prandtl numbers greater than unity, there is essentially little difference between the two.

Turbulence Model

To compute the velocity profile, Eq. 4, and the Nusselt number, Eq. 11 or 17, would require a specification of an appropriate turbulence model for ϵ_M and ϵ_H in the liquid film.

A survey shows that the modified van Driest model used by Limberg (1973) and Seban and Faghri (1976) seems to be a promising one. In this work, we have further modified their model to include the effects of interfacial shear through the variable shear stress term and included a van Driest type turbulent Prandtl number model to account for the difference in ϵ_M and ϵ_H as have been experimentally observed in single-phase flow. The eddy diffusivity distribution for momentum is divided into two regions, for $y/\delta \leqslant 0.6$,

$$\frac{\epsilon_M}{\nu} = -0.5 + 0.5\left\{1 + 0.64y^{+2}\frac{\tau}{\tau_w} \times\left\{1 - \exp\left(\frac{-y^+(\tau/\tau_w)^{1/2}}{A^+}\right)\right\}^2 f^2\right\}^{1/2} \tag{18}$$

where $\tau/\tau_w = 1 - s^3y^+/\delta^+$, $A^+ = 25.1$ and $f = \exp(-1.66(1 - \tau/\tau_w))$ is a damping factor. Equation 18 differs from the model used by Limberg and Seban and Faghri in that the shear stress and

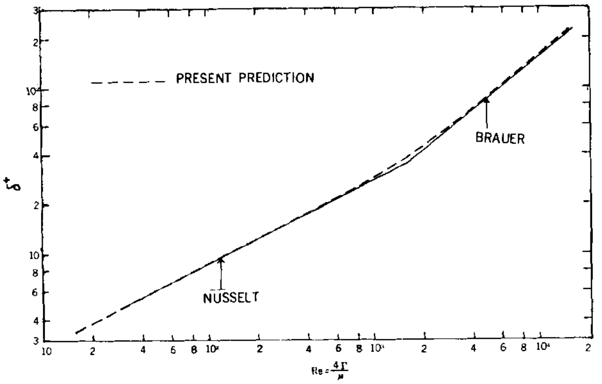

Figure 1. Average film thickness prediction for nonsheared film.

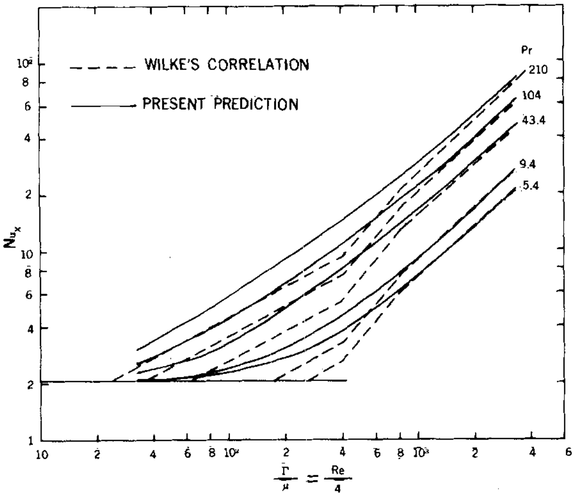

Figure 2. Prediction of Nu_x vs. Γ/μ as compared to Wilke's correlation (1962) for heating.

damping factor terms are modified to include the effect of interfacial shear. For $y/\delta > 0.6$, the eddy diffusivity is taken as constant and equal to its value at $y/\delta = 0.6$,

$$\epsilon_M/\nu = \epsilon_M/\nu\big|_{y/\delta=0.6} = \text{constant} \tag{19}$$

The turbulent Prandtl number is evaluated from Cebeci's modification of the van Driest model and is further modified in this work to include the effect of variable shear:

$$Pr_t = \frac{\epsilon_M}{\epsilon_H} = \frac{1-\exp(-y^+(\tau/\tau_w)^{1/2}/A^+)}{1-\exp(-y^+(\tau/\tau_w)^{1/2}/B^+)} \tag{20}$$

where B^+ is given by Habib and Na (1974) as

$$B^+ = Pr^{-1/2}\sum_{i=1}^{5} C_i(\log_{10}Pr)^{i-1}, \tag{21}$$

where $C_1 = 34.96$, $C_2 = 28.97$, $C_3 = 33.95$, $C_4 = 6.33$, $C_5 = -1.186$. Equation 20 indicates that Pr_t is larger than 1 near the wall but approaches 1 as $y^+ \rightarrow \delta^+$. Equation 20 differs from the model used by Seban and Faghri in that the shear stress term now includes the effect of interfacial shear and we use the Cebeci coefficient of 1.0 instead of 0.9. The above model reduces readily to the nonsheared film case when $s = 1$.

Calculation Procedure

First select a typical value of τ_i^* and a value of δ^+, substitute into Eq. 3 and solve for s by a root-finding procedure. The quantity ϵ_M/ν is then calculated from Eqs. 18 and 19, and together with s are substituted into Eq. 4 to solve for the velocity profile, $u^+(y^+)$, by a fourth-order Runge Kutta method. Then the velocity profile is integrated to give the film Reynolds number using a trapezoidal rule. Since $\delta^* = s\delta^{+2/3}$, we can obtain a plot of the dimensionless film thickness, δ^*, vs. Re for different values of τ_i^*. To calculate heat transfer results, Eqs. 18–21 are used to evaluate ϵ_H/ν and the above results for δ^+, Re and $u^+(\eta)$ are substituted into Eq. 11 or 17 to calculate the Nusselt number or the dimensionless heat transfer coefficient, h_x^*, as a function of Re for different values of τ_i^*. When there is no interfacial shear, $\tau_i^* = 0$ and $s = 1$, the calculation procedure is about the same.

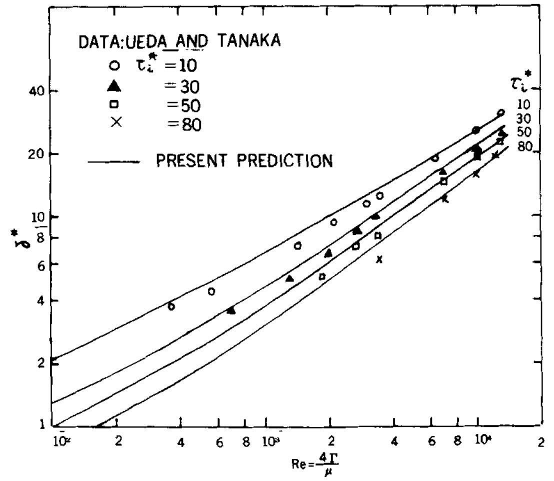

Figure 3. Average film thickness prediction as compared to data of Ueda and Tanaka (1974) for heating with interfacial shear.

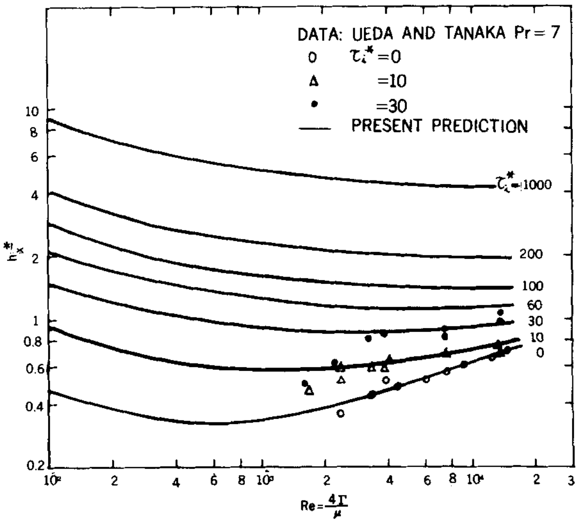

Figure 4. Prediction of h_x^* vs. Re as compared to data of Ueda and Tanaka (1974) for heating with interfacial shear.

RESULTS AND DISCUSSION

Heating

With no interfacial shear, the dimensionless film thickness, δ^+, as a function of Re was calculated and found to agree closely (within 5%) with the Nusselt equation in the laminar and wavy-laminar flow region of $Re < 1{,}600$ and with the Brauer equation in the turbulent flow region of $Re > 1{,}600$, Figure 1. In Figure 2, our predicted asymptotic Nusselt numbers are compared with Wilke's correlations. Comparisons were also made with Limberg's (1973) and Ishigai et al.'s (1974) predictions. It is found that the best agreement is obtained between our prediction and Wilke's correlations for $\Gamma/\mu > 600$. The Limberg and Ishigai predictions were

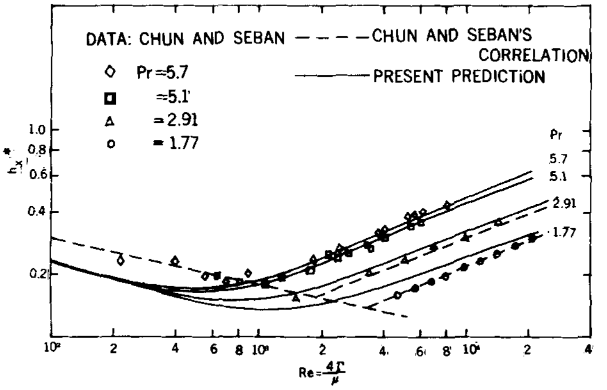

Figure 5. Prediction of h_x^* vs. Re as compared to evaporation data of Chun and Seban (1971).

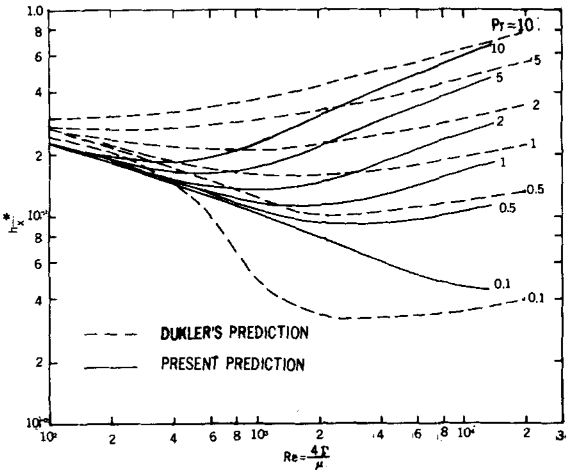

Figure 6. Prediction of h_x^* vs. Re as compared to Dukler's prediction (1960, 1961b) for condensation at various Pr.

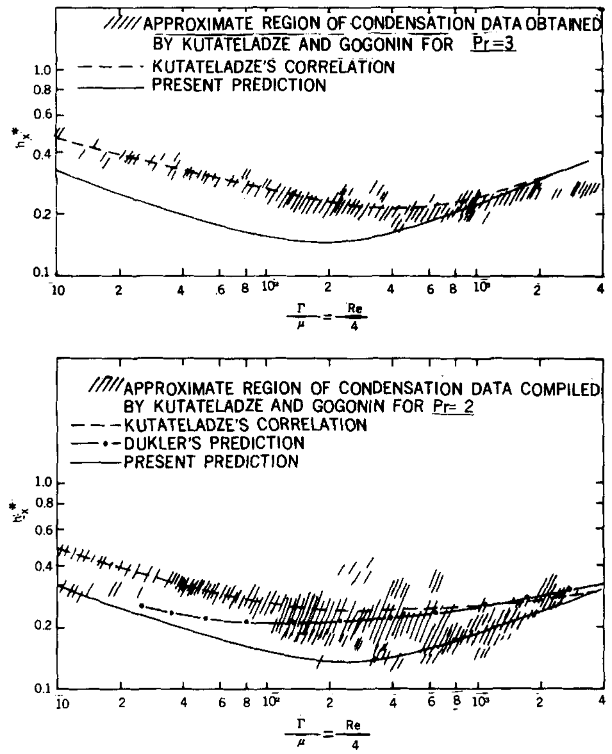

Figure 7. Prediction of h_x^* vs. Re as compared to condensation data compiled by Kutateladze and Gogonin (1979).

both higher than our predictions and Wilke's correlations. For $\Gamma/\mu < 600$, all three models overestimate the Nusself number.

Ueda and Tanaka (1974) obtained some data on film thickness and heat transfer coefficients for heating of a turbulent falling film with cocurrent gas interfacial shear. Dimensionless film thickness δ^* was obtained as a function of Re and τ_i^*. A comparison in Figure 3 shows that our prediction is slightly higher than their data, but for Re > 4000, the agreement becomes increasingly close. Satisfactory agreement is also obtained between our predictions and their data on dimensionless heat transfer coefficients at $Pr = 7$, Figure 4. At large τ_i^*, h_x^* is much more dependent on τ_i^* than on Re.

Evaporation

With negligible interfacial shear, the film thickness prediction is the same as that shown in Figure 1 when the evaporation rate is small and the film thickness can be considered as constant. Our heat transfer prediction is compared with the evaporation data of Chun and Seban (1969, 1971) in Figure 5. Good agreement is obtained for $Pr = 5.1$ and 5.7, but the prediction is slightly higher than the data for $Pr = 2.91$ and 1.77. Seban and Faghri (1976) have also compared their model prediction with the data of Chun and Seban and found that good agreement is obtained for $Pr = 1.77$, but the prediction is slightly lower than the data for $Pr = 5.7$. Our prediction is, therefore, slightly higher than their prediction because they have used the gas absorption eddy diffusivity in the interface region.

We have also attempted to use the gas absorption eddy diffusivity in a very small region near the interface as Seban and Faghri have done. We find that the prediction values and the slope of the prediction curve are both lowered. While the agreement with the data for $Pr = 2.91$ and 1.77 becomes better, the prediction is now lower than the data for $Pr = 5.1$ and 5.7. We feel that either model is satisfactory as a result of comparison with data. Therefore, the inclusion of the interface damping eddy diffusivity does not improve the prediction to such an extent that it is felt warranted.

Condensation

With no interfacial shear, the calculated results of δ^+ as a function of Re are the same as in heating or evaporation. In Figure 6, our heat transfer prediction is compared with Dukler's prediction (1960, 1961b), who used Deissler's eddy viscosity model for $y^+ \leq 20$ and von Karman's model for $y^+ > 20$, and the turbulent Prandtl number is set to be 1. Our prediction is lower than Dukler's prediction for $Pr = 0.5$ to 10, but is higher for $Pr = 0.1$. Our prediction approaches Dukler's prediction at increasing Reynolds number. The slopes of Dukler's curves are smaller than our curves, and this means a lesser dependence on Reynolds number.

Kutateladze and Gogonin (1979) have compiled extensive data for $Pr = 2$ and 3 from various authors. We have indicated in Figure 7 the approximate region of condensation data compiled by them. For $\Gamma/\mu > 600$, the present prediction seems to fit reasonably the data but the prediction is slightly lower than Kutateladze's or Dukler's prediction. For $\Gamma/\mu < 600$, the present prediction is much lower than the data. Interfacial shear exerted by the high velocity steam is an important factor in determining the heat transfer rate in film condensation. In Figure 8, calculated dimensionless film thickness δ^* as a function of Re and τ_i^* are found to conform well to, although slightly higher than, the data of Ueda et al. (1974). The present results are very close to the calculations of δ^* using the Dukler model.

In Figure 9, our prediction is also seen to agree well with the low interfacial shear condensation data of Blangetti and Schlunder (1978) for $\Gamma/\mu > 600$. Some high interfacial shear condensation data were obtained by Ueda et al. (1974). They are replotted in Figure 10 and compared with our prediction and the predictions of Rohsenow et al. (1956) and Hubbard et al. (1976). Rohsenow et

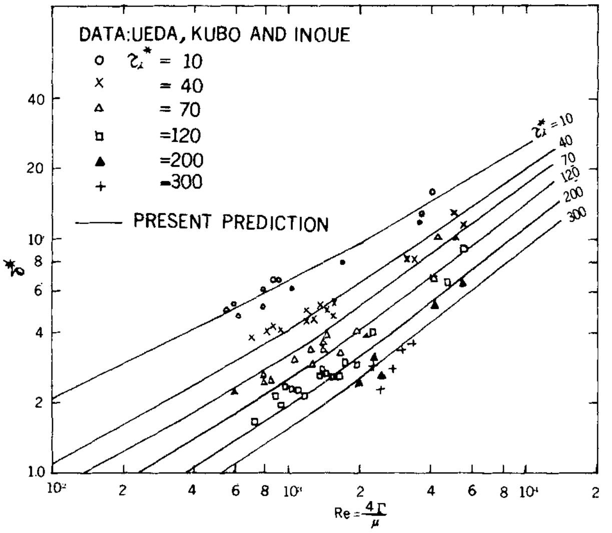

Figure 8. Average film thickness prediction as compared to data of Ueda et al. (1974) for condensation with interfacial shear.

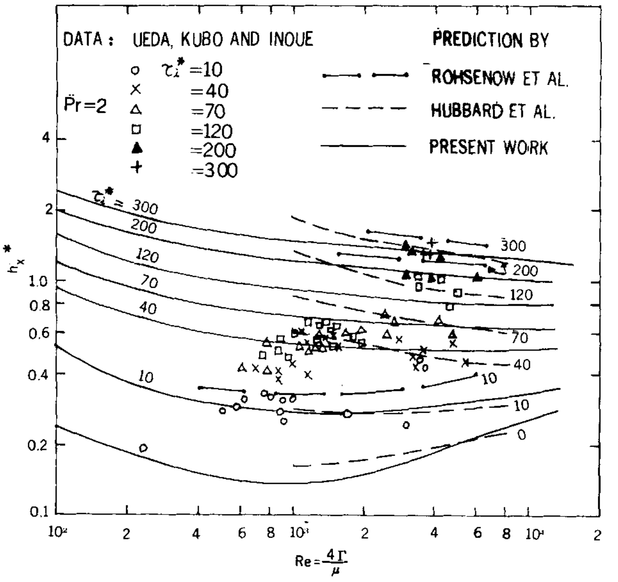

Figure 10. Prediction of h_x^* vs. Re as compared to data of Ueda et al. (1974) for condensation with interfacial shear.

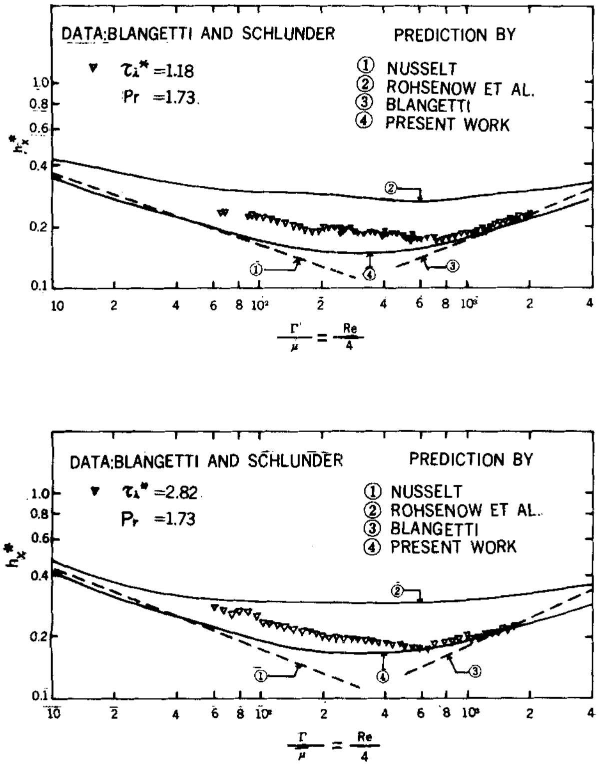

Figure 9. Prediction of h_x^* vs. Γ/μ as compared to data of Blangetti and Schlunder (1978) for condensation with interfacial shear.

al.'s prediction seems to be too high. Hubbard et al.'s prediction and our prediction are both in satisfactory agreement with the data in view of the large scattering of the data.

Hubbard et al. have also used an empirical gas absorption eddy diffusivity deduced from Chung's data (1974) in the interface region, but this does not seem to offer any advantage or better accuracy. In fact, their model gives a lower Reynolds number dependence when compared with the evaporation data of Chun and Seban (1971). The addition of an interface eddy diffusivity in the present model would have an adverse effect because it decreases the heat transfer coefficient such that the prediction would become lower than the condensation data.

The above comparisons with some literature data obtained on film heating, evaporation and condensation processes show that the present model, on the average, gives a good prediction of both the film thickness and asymptotic heat transfer coefficient in the turbulent flow region of $Re > 2{,}400$. Moreover, Yih and Chen (1982) have shown analytically that the present turbulence model predicts the heat transfer coefficient of heating or evaporation in the thermal entrance region of a falling film very well as compared to a finite difference numerical solution of Faghri (1976) and Seban and Faghri (1976). Also, the agreement between the present calculated results and data serves to reemphasize one point: the specification of a gas absorption eddy diffusivity for the interface region of film heating, evaporation or condensation is unnecessary in view of the unknown thickness of this region.

For $Re < 2{,}400$, the model still predicts the film thickness well but often over- or under-estimates the heat transfer coefficient in the laminar and wavy-laminar flow regions as can be seen from Figures 2, 4, 5, 7, 9 and 10. This is because the turbulence models are, in principle, not applicable to these regions. Moreover, it is noted that most of the models previously proposed, Table 1, did not account for the variable shear stress in the liquid film and so give predictions of average film thickness and heat transfer coefficient which are higher than the data. This illustrates the importance of including the effect of variable shear stress in the turbulence model. Also, the damping factor f is necessary as shown by Limberg, Seban and Faghri. Otherwise, the prediction of film thickness and heat transfer coefficient will be too high.

The turbulent Prandtl number incorporated is theoretically sound as has been shown in application to single-phase flow. If Pr_t is set equal to 1 as have been done by Dukler and some others, the heat transfer predictions will be too high as compared with the data. Our calculations using $Pr_t = 1$ also show this to be true. At low Pr commonly encountered in heating, evaporation and condensation, the effect of Pr_t is very important and cannot be neglected.

Chun and Seban (1971) have compared their evaporation data with Dukler's predictions (1960, 1961b). Dukler's predictions were found to be too high and gave a slope that did not agree with the data in the turbulent flow region. In contrast, our predictions as shown in Figure 5 give a much better agreement with Chun and Seban's data, and the slope also follows their experimental trend. Dukler's model will most probably give a much higher heat transfer prediction than ours because his $Pr_t = 1$ and he has not included a damping factor f. Therefore, if his model is used, his prediction curve will lie above the data in our Figures 1–10 and so is not as good as our prediction. At large interfacial shear, our predictions are close to Dukler's.

NOTATION

A^+ = constant = 25.1
B^+ = a van Driest parameter
C_i = constants
f = a damping factor
g, g_c = gravitational acceleration constant, m/s^2; proprtional factor, kg·m/N·s^2
h_x = heat transfer coefficient, W/m^2·K
k = thermal conductivity, W/m·K
Nu_x = Nusselt number = $h_x\delta/k$
Pr, Pr_t = Prandtl number; turbulent Prandtl number = ϵ_M/ϵ_H
q_w = wall heat flux, W/m^2
Re = film Reynolds number = $4\Gamma/\mu$
s = $((\rho\delta g/g_c)/(\tau_i + \rho\delta g/g_c))^{1/3}$
T, T_{sat} = liquid temperature at saturation
u, u^*, u^+ = liquid velocity; friction velocity, $(\tau_w g_c/\rho)^{1/2}$; u/u^*
x, x^* = axial film distance; $x\alpha/\delta^2 u^*$, dimensionless
y, y^+ = distance measured from the wall; yu^*/ν

Greek Letters

α = thermal diffusivity, m^2/s
Γ = mass flow rate per unit periphery, kg/m·s
$\delta, \delta^*, \delta^+$ = film thickness; $\delta/(\nu^2/g)^{1/3}$; $\delta u^*/\nu$
η = y/δ, or y^+/δ^+
$\theta, \theta_w, \theta_m$ = dimensionless temperature, $(T - T_{in})/(q_w\delta/k)$ for heating and $(T - T_{sat})/(q_w\delta/k)$ for evaporation or condensation; at wall; at bulk mean
ϵ_M, ϵ_H = eddy diffusivity for momentum; for heat, m^2/s
σ = surface tension, N/m
μ = liquid viscosity, kg/m·s
ν = kinematic viscosity, m^2/s
ρ = liquid density, kg/m^3
τ, τ_w, τ_i = shear stress; at wall; at interface, N/m^2
τ_i^* = $\tau_i g_c/\rho(\nu g)^{2/3}$, dimensionless

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